Modeling and Output-Only Identification of Linear Damped Structures

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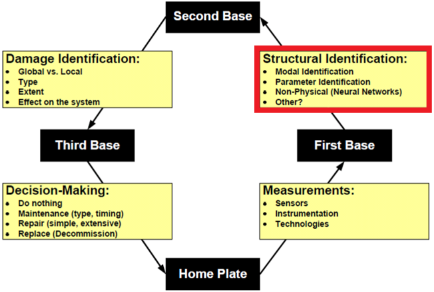
Motivation

Structural Health Monitoring



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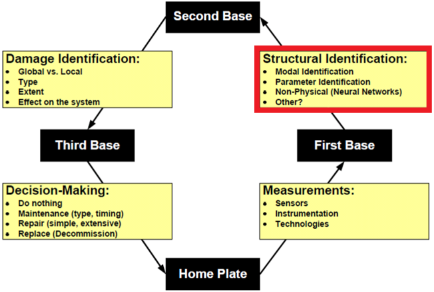

Structural Identification

- Black-Box models;
- Modal models;
- Physical models.

Output-Only

It is not always feasible to:

- Measure the input acting on a structure;
- Excite the structure with input overcoming the noise.

Ettouney, M., Alampalli, S. (2011), "Infrastructure Health in Civil Engineering"
Alvin, K.F., Robertson A.N., Reich, G.W., Park, K.C. (2003), "System Identification: From Reality to Models"
Cunha, A., Caetano, E., Magalhaes, F., Moutinho, C. (2005), "From Input-Output to Output-Only Modal Identification of Civil Engineering Structures"

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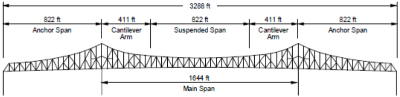
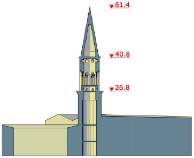


Aktan E., Catbas F.N. (2000), "Real-Time Bridge Health Monitoring for Managment"



Gentile C., Ubertini F. (2014), "Dynamic investigation of the San Pietro bell-tower in Perugia"

Objectives

Main Objective

Definition of an output-only identification methodology for linear damped structures, consisting in three steps:

- Identification of black-box models;
- Definition of modal models (**viscous** and **non-viscous** damping);
- Definition of physical models through Finite Element Model Updating (FEMU).

Objectives

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Field of Application

Enlarge the field of application of output-only identification to structures with non-uniform damping properties.

Models

Black-Box Models

$$\dot{\mathbf{z}}(t) = \mathbf{A}\,\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{z}(t) + \mathbf{D}\mathbf{u}(t)$$

Models

Black-Box Models

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To identify

Unknown

Models

Black-Box Models

$$
\begin{aligned}
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\end{aligned}
$$

To identify

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Models

Black-Box Models

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To identify

Unknown

Modal Models

$$\mathbf{\Lambda} = diag(\lambda_1, \lambda_1^*, ..., \lambda_N, \lambda_N^*, \lambda_p); \mathbf{\Psi} = (\psi_1, \psi_1^*, ..., \psi_N, \psi_N^*, \psi_p)$$

Models

Black-Box Models

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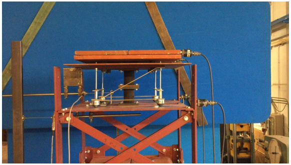
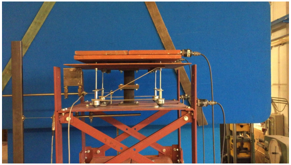
Physical Models

$$(\mathbf{M}, \mathbf{K}, \mathbf{L}) = f(\mathbf{\Lambda}, \mathbf{\Psi})$$

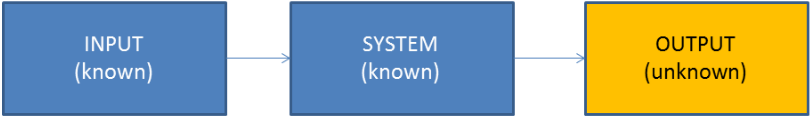
Case Studies



Case Studies



Direct Problem and Non-Viscous Damping

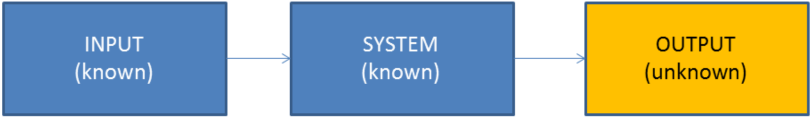


It is addressed the direct problem relative to linear structures, whose dissipative forces are expressed as:

$$f_L(t) = \int_{-\infty}^{t} \mathcal{G}(t-\tau)\,\dot{\Delta}(\tau)d\tau$$

Damping Kernel function

Direct Problem and Non-Viscous Damping



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Damping Kernel function

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \int_{-\infty}^{t} \mathbf{G}(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{0}$$

$$\mathbf{x}(0) = \mathbf{x}_0; \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0$$

Adhikari, S. (2010), "Damping Models for Structural Vibration". Ph.D. Trinity College, Cambridge.

Non-Viscous Damping - Maxwell-Weichert Model

A frequently used model for $\mathcal{G}(t-\tau)$ is the Maxwell-Weichert model:

$$\mathcal{G}(t) = \sum_{l=1}^{n_w} c_l \, \mu_l \, e^{-\mu_l t} \qquad \mu_l = k_l / c_l$$

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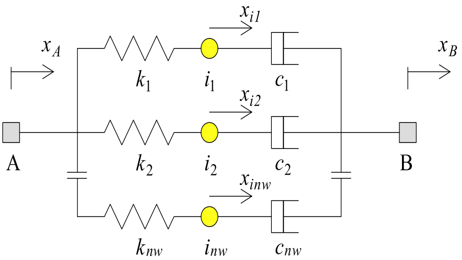
N dynamic DOFs

$p = \sum n_w$ internal DOFs



Wagner, N., Adhikari, S. (2003), "Symmetric State-Space Method for a Class of Nonviscously Damped Systems"

Non-Viscous Damping Vs Inverse Problem

Symmetric First-Order Formulation

$$\mathbf{E}\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t)$$
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$$\mathbf{z}(t) = \begin{pmatrix} \mathbf{x}_d(t) \\ \mathbf{x}_i(t) \\ \dot{\mathbf{x}}_d(t) \end{pmatrix} ; \mathbf{E} = \begin{pmatrix} \mathbf{L}_{dd} & \mathbf{L}_{di} & \mathbf{M}_{dd} \\ \mathbf{L}_{di}^T & \mathbf{L}_{ii} & \mathbf{0} \\ \mathbf{M}_{dd} & \mathbf{0} & \mathbf{0} \end{pmatrix} ; \mathbf{A} = - \begin{pmatrix} \mathbf{K}_{dd} & \mathbf{K}_{di} & \mathbf{0} \\ \mathbf{K}_{di}^T & \mathbf{K}_{ii} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{M}_{dd} \end{pmatrix}$$

$$dd \rightarrow (N \times N)$$
$$di \rightarrow (N \times p)$$
$$ii \rightarrow (p \times p)$$

Reggio, A., De Angelis, M., Betti, R. (2013), "A State-Space Methodology to Identify Modal and Physical Parameters of Non-Viscously Damped Systems"

Non-Viscous Damping - Eigenvalue Problem

Eigenvalue Problem

$$\mathbf{E\Psi\Lambda} = \mathbf{A\Psi}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \mathbf{\Lambda}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_i \end{pmatrix} = diag(\lambda_{d1}, \lambda_{d1}^*, ..., \lambda_{dN}, \lambda_{dN}^*, \lambda_{i1}, ..., \lambda_{ip})$$

$$\mathbf{\Lambda}_d \in \mathbb{C}^{2N\times 2N} \qquad \mathbf{\Lambda}_i \in \mathbb{R}^{p\times p}$$

$$\mathbf{\Psi} = \begin{pmatrix} \mathbf{\Psi}_d \\ \mathbf{\Psi}_i \\ \mathbf{\Psi}_d\mathbf{\Lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{\Psi}_{dd} & \mathbf{\Psi}_{di} \\ \mathbf{\Psi}_{id} & \mathbf{\Psi}_{ii} \\ \mathbf{\Psi}_{dd}\mathbf{\Lambda}_d & \mathbf{\Psi}_{di}\mathbf{\Lambda}_i \end{pmatrix}$$

$$\mathbf{\Psi}_d \in \mathbb{C}^{N\times 2N+p} \qquad \mathbf{\Psi}_i \in \mathbb{R}^{p\times 2N+p}$$

Reggio, A., De Angelis, M., Betti, R. (2013), "A State-Space Methodology to Identify Modal and Physical Parameters of Non-Viscously Damped Systems"

Non-Viscous Vs Viscous Damping

$\mathcal{G}(t) = c_0\delta(t) \rightarrow$ Viscous damping model

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- $\mu_d \rightarrow \infty$, i.e. $k_d \rightarrow \infty$, as $\mathcal{G}(t)$ tends to have no memory;

Non-Viscous Vs Viscous Damping

$\mathcal{G}(t) = c_0\delta(t) \rightarrow$ Viscous damping model

- $\mu_d \rightarrow \infty$, i.e. $k_d \rightarrow \infty$, as $\mathcal{G}(t)$ tends to have no memory;
- The internal DOFs disappear in the modeling, only the dynamic DOFs remain (the physical matrices become of dimensions $N \times N$).

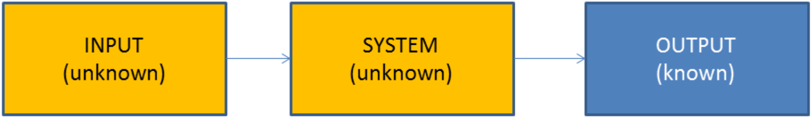
The symmetric first-order problem becomes:

$$\begin{pmatrix} \mathbf{L} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\Psi\Lambda} \end{pmatrix} \boldsymbol{\Lambda} = \begin{pmatrix} -\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\Psi\Lambda} \end{pmatrix}$$

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_d \in \mathbb{C}^{2N\times 2N}; \boldsymbol{\Psi} = \begin{pmatrix} \boldsymbol{\Psi}_d \\ \boldsymbol{\Psi}_d\boldsymbol{\Lambda} \end{pmatrix} \in \mathbb{C}^{2N\times 2N}$$

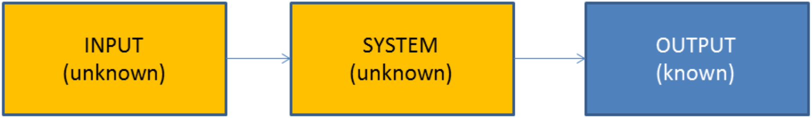
De Angelis, M., Lus, H., Betti, R., Longman, R.W. (2002), "Extracting Physical Parameters of Mechanical Models from Identified State Space Representations"

Inverse Problem - First Step



Identify a black-box model (the order n, the state matrix $\mathbf{A}_d$, the output matrix $\mathbf{C}_d$) by Data-Driven Stochastic Subspace Identification (**Data-SSI**)

Inverse Problem - First Step



Identify a black-box model (the order n, the state matrix $\mathbf{A}_d$, the output matrix $\mathbf{C}_d$) by Data-Driven Stochastic Subspace Identification (**Data-SSI**)

- The computational effort is reduced because it uses the response data directly;
- It involves consolidated linear algebra tools (SVD and QR factorization);
- Non-linear computations are avoided.

De Moor, B., Van Overschee, P. (1996), "Subspace Identification for Linear Systems"

Data-SSI

Steps

$$\mathbf{x}_{k+1} = \mathbf{A}_d\mathbf{x}_k + \mathbf{w}_k$$
$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k$$

Data-SSI

Steps

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{w}_k$$
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$$\mathbf{H} = \begin{pmatrix} \mathbf{Y}_p \\ \mathbf{Y}_f \end{pmatrix}$$

Data-SSI

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$$\mathbf{H} = \begin{pmatrix} \mathbf{Y}_p \\ \mathbf{Y}_f \end{pmatrix}$$

$$\mathbf{P} = \mathbf{W}_1(\mathbf{Y}_f/\mathbf{Y}_p)\mathbf{W}_2$$

Data-SSI

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$$\mathrm{SVD}(\mathbf{P}) \rightarrow n, \mathbf{O}$$

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$$\mathbf{O} \rightarrow \mathbf{A}_d, \mathbf{C}$$

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Lacks in Literature

- Rules for user-defined parameters i, j, g, h
- Relationship between the parameters and the sampling frequency fs
- Influence of $\mathbf{W}_1, \mathbf{W}_2$
- Non-stationary seismic excitation

Parameters in Data-SSI

$$
\mathbf{H} = \left(\frac{\mathbf{Y}_p}{\mathbf{Y}_f}\right) =
\begin{array}{c}
j \\
\begin{pmatrix}
\mathbf{y}(0) & \cdots & \mathbf{y}(j-1) \\
\mathbf{y}(1) & \cdots & \mathbf{y}(j) \\
\vdots & \ddots & \vdots \\
\mathbf{y}(g-1) & \cdots & \mathbf{y}(g+j-2) \\
\mathbf{y}(i-h) & \cdots & \mathbf{y}(i-h+j-1) \\
\mathbf{y}(i-h+1) & \cdots & \mathbf{y}(i-h+j) \\
\vdots & \ddots & \vdots \\
\mathbf{y}(i-1) & \cdots & \mathbf{y}(i+j-2)
\end{pmatrix}
\end{array}
\begin{array}{l}
\\
m_x g \\
\\
m_x h
\end{array}
\quad m_x i
$$

$\mathbf{Y}_p \in \mathbb{R}^{m \cdot g \times j}$ are the past outputs

$\mathbf{Y}_f \in \mathbb{R}^{m \cdot h \times j}$ are the future outputs

Parameters in Data-SSI

$$\mathbf{H}=\left(\frac{\mathbf{Y}_p}{\mathbf{Y}_f}\right)=\begin{array}{cc} \overset{j}{\longleftrightarrow} & \\ \begin{pmatrix} \mathbf{y}(0) & \cdots & \mathbf{y}(j-1) \\ \mathbf{y}(1) & \cdots & \mathbf{y}(j) \\ \vdots & \ddots & \vdots \\ \mathbf{y}(g-1) & \cdots & \mathbf{y}(g+j-2) \\ \mathbf{y}(i-h) & \cdots & \mathbf{y}(i-h+j-1) \\ \mathbf{y}(i-h+1) & \cdots & \mathbf{y}(i-h+j) \\ \vdots & \ddots & \vdots \\ \mathbf{y}(i-1) & \cdots & \mathbf{y}(i+j-2) \end{pmatrix} & \begin{array}{l} m \times g \\ \\ m \times h \end{array} \quad m \times i \end{array}$$

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- *m*: Measurement points

$\mathbf{Y}_p \in \mathbb{R}^{m \cdot g \times j}$ are the past outputs

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Parameters in Data-SSI

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\overset{j}{\longleftrightarrow} & \\
\begin{pmatrix}
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\end{array}
\end{array}
$$

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- *i*: Number of rows
- *j*: Number of columns

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- m: Measurement points
- i: Number of rows
- j: Number of columns
- h: Number of rows of $\mathbf{Y}_f$
- g: Number of rows of $\mathbf{Y}_p$

$\mathbf{Y}_p \in \mathbb{R}^{m\cdot g\times j}$ are the past outputs

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$$g+h=i$$

Priori, C., De Angelis M. (2015), "Output-Only Identification of Base Excited Structures Using Subspace Methods With Asymmetric Partitions".

Rules for parameters in Data-SSI

Parameter i

$$i(f_1/fs) > n_c \longrightarrow n_c = 1 \longrightarrow i_{min} = \frac{fs}{f_1}$$

- f_1: Lower frequency
- n_c: Number of cycles of f_1

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$$g_{min,t} = n/m$$

$$\boxed{g_{min} \simeq 10\frac{n}{m}}$$

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Parameter j

$$j = s - i + 1$$

$$j_{min} = n_c \frac{1}{\hat{g}} \frac{fs}{f_1}$$

Weighting Matrices in Data-SSI

The influence of four different choices of the weighting matrices $\mathbf{W}_1$ and $\mathbf{W}_2$, coming out from multivariate analysis, is investigated:

- Partial Least Squares (PLS)
- Canonical Correlation Analysis (CCA)
- Multiple Linear Regression (MLR)
- Enhanced Canonical Correlation Analysis (ECCA)

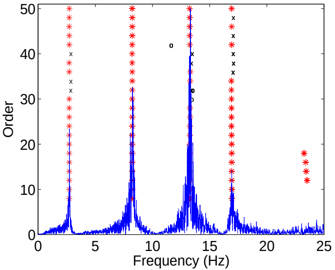
Priori, C., De Angelis M., Reggio, A. (2013), "Identificazione ad input incognito di ponti isolati con HDRB mediante risposte ad azioni sismiche"
Hong, A.L. (2010), "Weighting Matrices and Model Order Determination in Stochastic System Identification for Civil Infrastructure Systems"

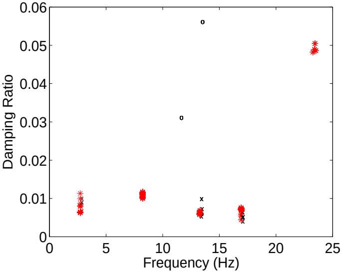
Second Step - Modal Models

$$\mathbf{C} = \mathbf{O}(1:m,:) \qquad \mathbf{A}_d = \mathbf{\Psi\Lambda\Psi}^T \rightarrow \mathbf{\Lambda} \in \mathbb{C}^{n\times n}$$

$$\mathbf{A}_d = \bar{\mathbf{O}}^{\dagger}\mathbf{O}(m+1:m\cdot h,:) \qquad \mathbf{\Phi} = \mathbf{C\Psi} \in \mathbb{C}^{m\times n}$$

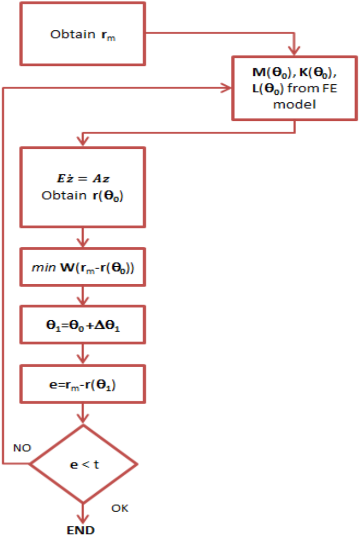
Due to noise, higher orders have to be selected and the identification of the modal model can be made by inspecting the stabilization diagrams (SD).





Priori, C., De Angelis M., Reggio, A. (2013), "Identificazione ad input incognito di ponti isolati con HDRB mediante risposte ad azioni sismiche"

Third Step - Sensitivity-based FEMU



$$\mathbf{e} = \mathbf{r}_m - \mathbf{r}(\boldsymbol{\theta})$$

$$\mathbf{F} = \mathbf{We}$$

$$min(\mathbf{F})$$

Minimization through sensitivity-based algorithm:

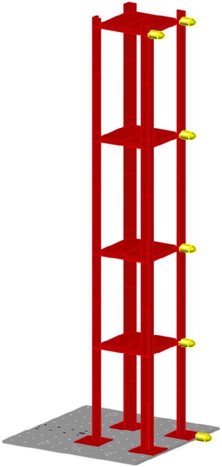
$$\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i + \Delta\boldsymbol{\theta}_i$$

$$\Delta\boldsymbol{\theta}_i = \mathbf{S}_i^{-1}\mathbf{W}\Delta\mathbf{r}_i$$

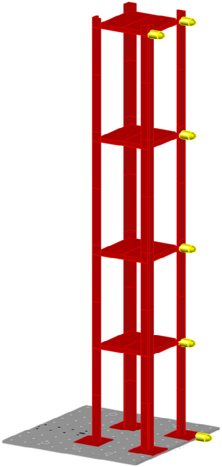
$$\mathbf{S}_i = \frac{\partial r_j}{\partial \theta_k}_{\boldsymbol{\theta}-\boldsymbol{\theta}_i}$$

Mottershead, J.E., Link, M., Friswell, M.I. (2011), "The Sensitivity Method in Finite Element Model Updating: A Tutorial".

Steel Frame Structure



Steel Frame Structure

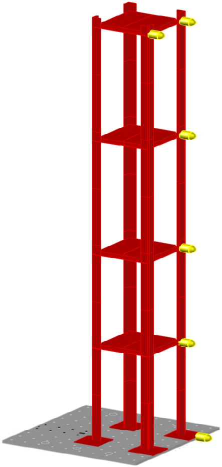


White Noise

Stationary signal with a large number of samples (large j)

- Influence of h and g
- Influence of i and j

Steel Frame Structure



White Noise

Stationary signal with a large number of samples (large j)

- Influence of h and g
- Influence of i and j

Seismic Records

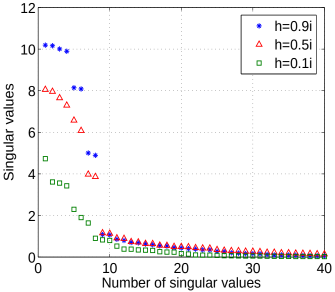
3 earthquakes of low PGA and reduced number of samples

- Influence of h and g
- Influence of $\mathbf{W}_1, \mathbf{W}_2$

Steel Frame Structure - White Noise

Variation of h

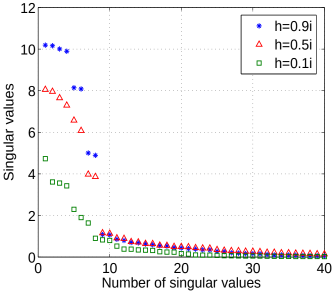
i	j	m	i_{min}	$\hat{h}_{min}$	$\hat{g}_{min}$	j_{min}
200	20005	4	184	0.9	0.1	1837

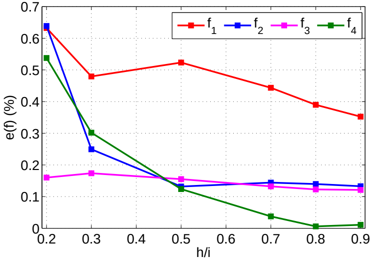


Steel Frame Structure - White Noise

Variation of h

i	j	m	i_{min}	$\hat{h}_{min}$	$\hat{g}_{min}$	j_{min}
200	20005	4	184	0.9	0.1	1837





Steel Frame Structure - White Noise

Variation of i

i	j	m	i_{min}	$\hat{h}_{min}$	$\hat{g}_{min}$	j_{min}
200	20005	4	184	0.9	0.1	1837
400	19805	4	184	0.46	0.05	3673

ξ	i	$e(\%)$
ξ_1	200	11.45
	400	17.00
ξ_2	200	35.49
	400	48.44
ξ_3	200	4.06
	400	8.94
ξ_4	200	0.44
	400	5.46

Steel Frame Structure - White Noise

Variation of i

i	j	m	i_{min}	$\hat{h}_{min}$	$\hat{g}_{min}$	j_{min}
200	20005	4	184	0.9	0.1	1837
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ξ	i	$e(\%)$
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	400	17.00
ξ_2	200	35.49
	400	48.44
ξ_3	200	4.06
	400	8.94
ξ_4	200	0.44
	400	5.46

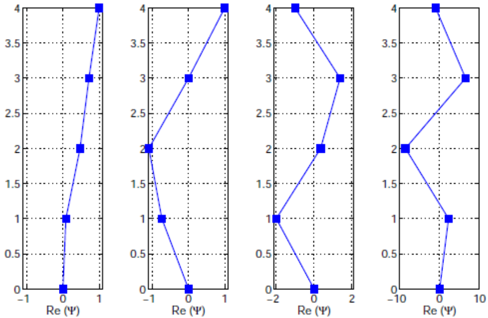
Variation of j

j	$e(\xi_1)$ (%)	$e(\xi_2)$ (%)	$e(\xi_3)$ (%)	$e(\xi_4)$ (%)
20005	11.45	35.49	4.06	0.44
5000	25.40	42.24	5.50	19.03
1500	31.12	62.51	13.43	46.79

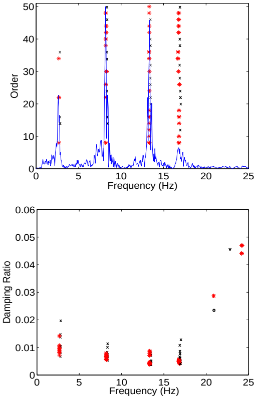
Steel Frame Structure - White Noise

Modal Parameters

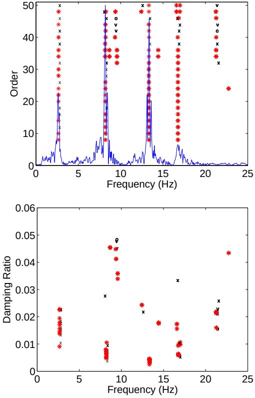
Case	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	ξ_1 (%)	ξ_2 (%)	ξ_3 (%)	ξ_4 (%)
REF	2.733	8.217	13.290	16.933	1.051	0.724	0.667	0.702
ECCA	2.727	8.228	13.274	16.935	0.943	1.122	0.641	0.699
	(0.22)	*(0.13)*	*(0.12)*	*(0.01)*	*(11.45)*	*(35.49)*	*(4.06)*	*(0.44)*



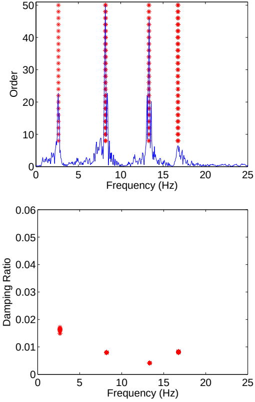
Steel Frame Structure - Seismic Records



h=0.1i

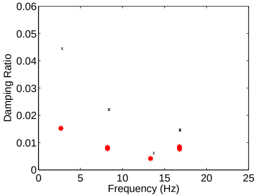


h=0.5i

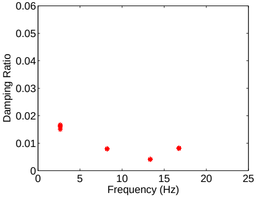


h=0.9i

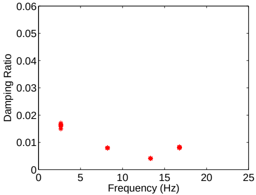
Steel Frame Structure - Weighting Matrices



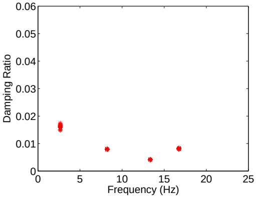
CCA



PLS



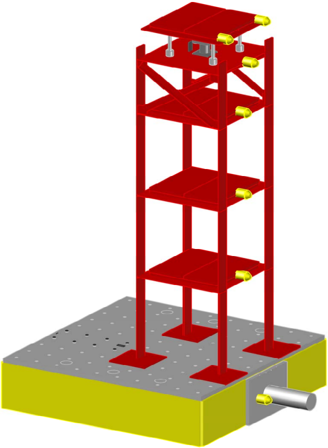
ECCA



MLR

Structure With Isolators

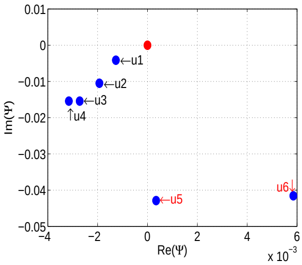
Structure With Isolators

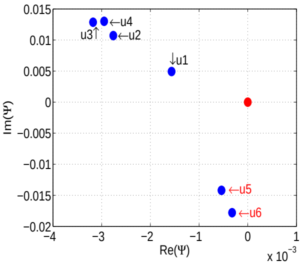


Structure With Isolators - Modal Parameters

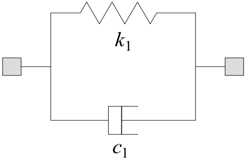
Case	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
REF	2.812	5.150	12.051	17.131	17.670
WNT0	2.765	5.146	12.024	17.121	17.645
	(1.60)	*(0.07)*	*(0.21)*	*(0.05)*	*(0.14)*

Case	ξ_1 (%)	ξ_2 (%)	ξ_3 (%)	ξ_4 (%)	ξ_5 (%)
REF	8.950	12.900	0.720	0.705	0.560
WNT0	8.420	12.870	0.560	0.610	0.610
	(5.92)	*(0.23)*	*(22.22)*	*(12.85)*	*(8.92)*



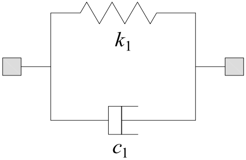


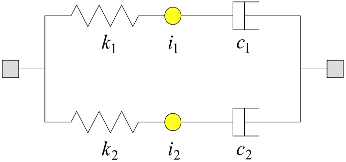
Structure With Isolators - Updating



Response	REF	UPD
f_{p1}	2.765	2.719
		(1.66)
f_{p2}	5.146	4.872
		(5.32)
ξ_{p1}	0.084	0.085
		(1.19)
ξ_{p2}	0.128	0.112
		(12.50)

Structure With Isolators - Updating



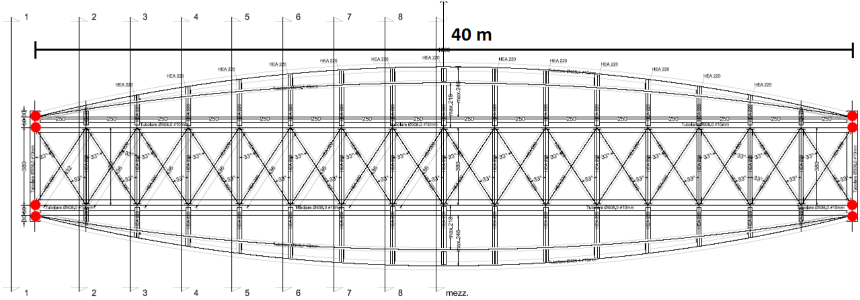


Response	REF	UPD
f_{p1}	2.765	2.719
		(1.66)
f_{p2}	5.146	4.872
		(5.32)
ξ_{p1}	0.084	0.085
		(1.19)
ξ_{p2}	0.128	0.112
		(12.50)

Response	REF	UPD
f_{p1}	2.765	2.761
		(0.14)
f_{p2}	5.146	5.135
		(0.21)
ξ_{p1}	0.084	0.084
		(0.36)
ξ_{p2}	0.128	0.129
		(0.54)

Footbridge over Velino

Footbridge over Velino



Footbridge over Velino

Motivation and Objectives

Performed Tests

Ambient vibration tests, walk-and-run tests

Motivation

- Presence of isolators
- Comfort problems

Motivation and Objectives

Performed Tests

Ambient vibration tests, walk-and-run tests

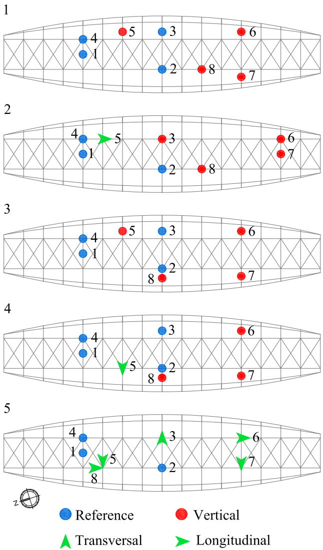
Motivation

- Presence of isolators
- Comfort problems

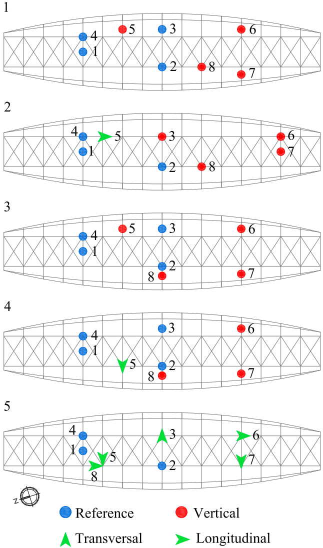
Objective

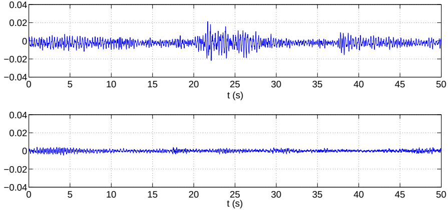
- Characterize the dynamic behaviour
- Define a FE models in agreement with the measured data

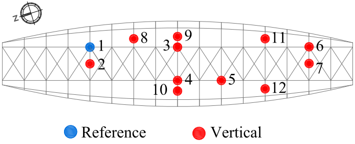
Setups



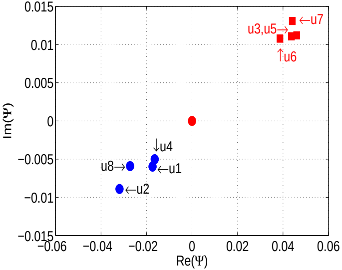
Setups


1
2
3
4
5
Reference
Vertical
Transversal
Longitudinal

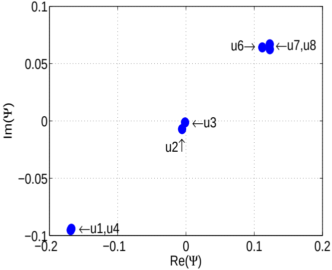

0.04
0.02
0
−0.02
−0.04
0 5 10 15 20 25 30 35 40 45 50
t (s)


Reference
Vertical

Identification of Black-Box and Modal Models



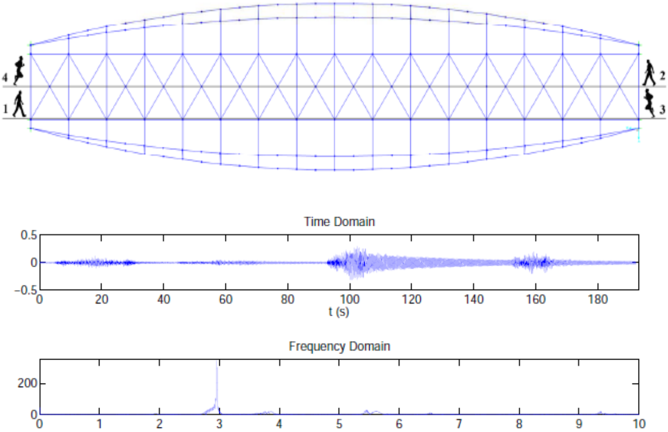
Setup 1 - 2.96 Hz



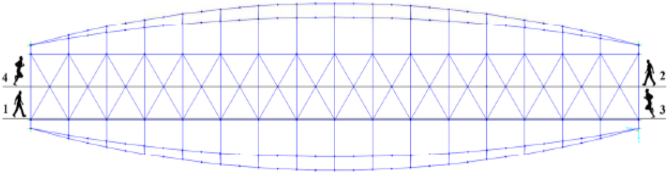
Setup 2 - 3.26 Hz

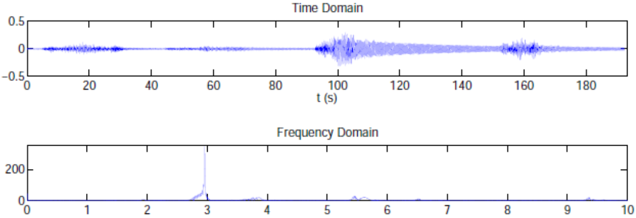
f (Hz)	2.302	2.964	3.277	3.781	6.479	6.805	9.373
ξ (%)	0.59	0.14	1.17	0.43	0.97	0.09	0.34

Comfort

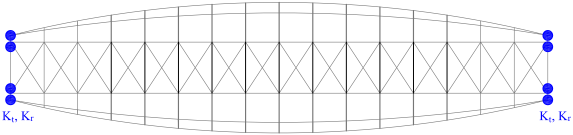

4
1
2
3
Time Domain
0.5
0
−0.5
0
20
40
60
80
100
120
140
160
180
t (s)
Frequency Domain
200
0
0
1
2
3
4
5
6
7
8
9
10

Comfort

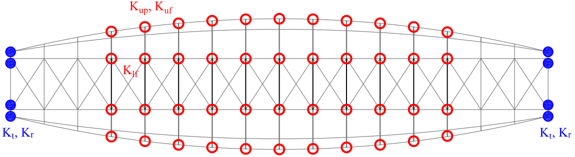

4
1
2
3

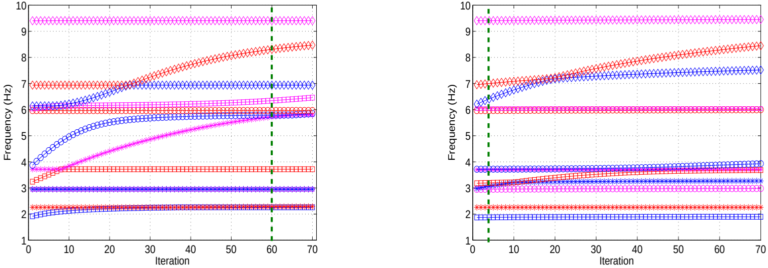

Time Domain
0.5
0
−0.5
0
20
40
60
80
100
120
140
160
180
t (s)
Frequency Domain
200
0
0
1
2
3
4
5
6
7
8
9
10

Physical Model - Parameters



Physical Model - Parameters

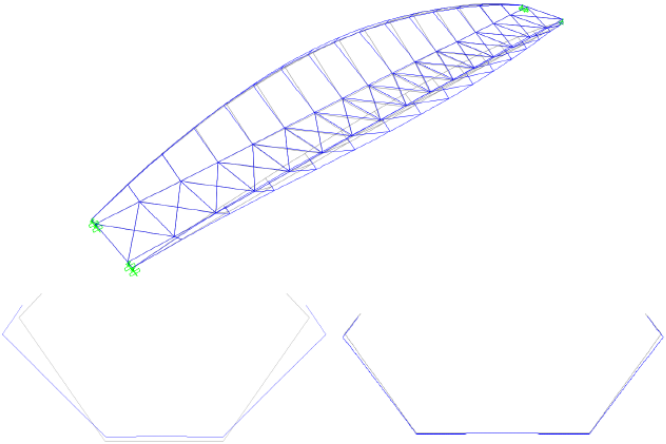




Physical Model - FEMU

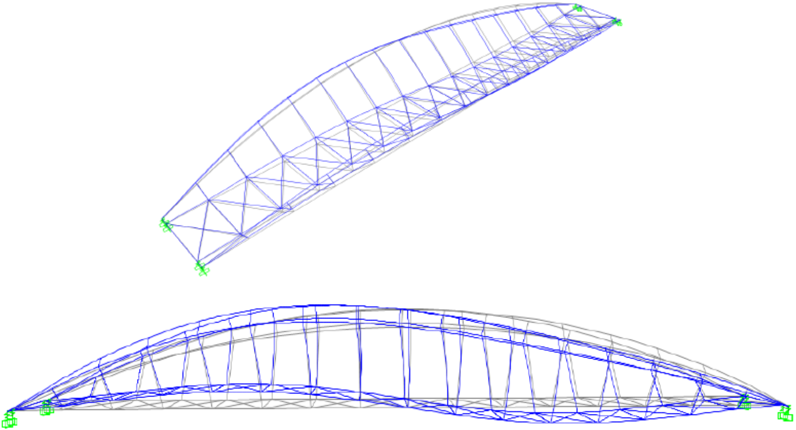
REF	ISO	INI		UPD	
f (Hz)	f (Hz)	f (Hz)	MAC	f (Hz)	MAC
2.302	-	2.325	0.941	2.322	0.962
	-	*(1.00)*		*(0.87)*	
2.964	-	2.984	0.870	2.981	0.886
	-	*(0.67)*		*(0.57)*	
3.277	3.041	3.164	0.922	3.255	0.959
	(7.21)	*(3.46)*		*(0.68)*	
3.782	3.572	3.771	0.940	3.775	0.966
	(5.74)	*(0.28)*		*(0.17)*	
6.479	5.756	6.369	0.870	6.465	0.880
	(11.96)	*(1.70)*		*(0.22)*	
6.805	5.968	6.999	0.921	7.000	0.942
	(12.30)	*(2.85)*		*(2.86)*	
9.373	-	10.457	0.840	10.508	0.834
	-	*(11.56)*		*(12.11)*	

Mode Shapes



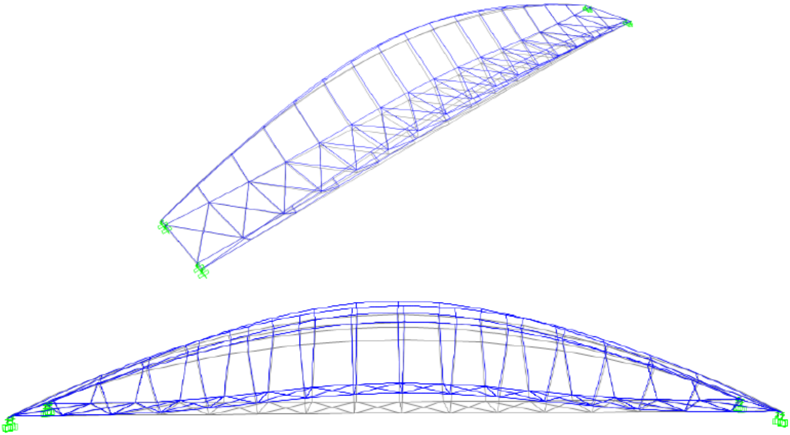
2.981 Hz

Mode Shapes



3.255 Hz

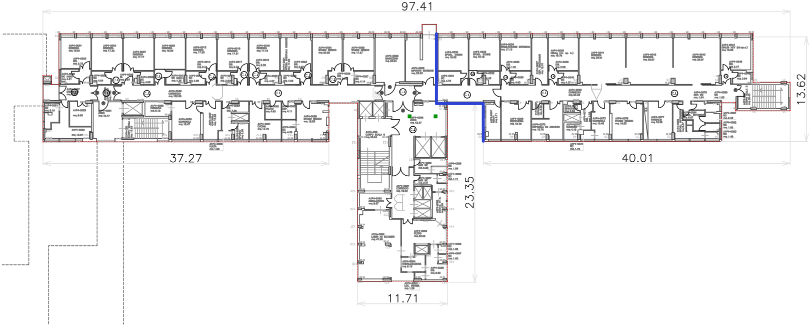
Mode Shapes



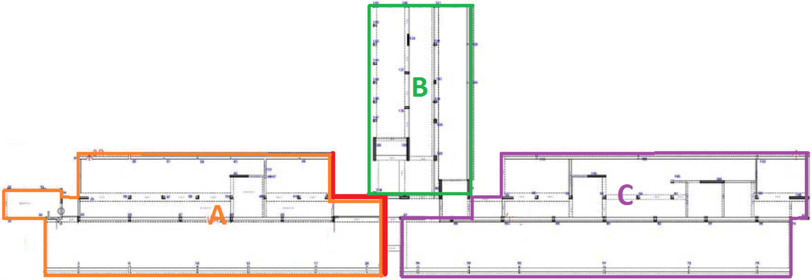
3.775 Hz

San Camillo de Lellis Hospital

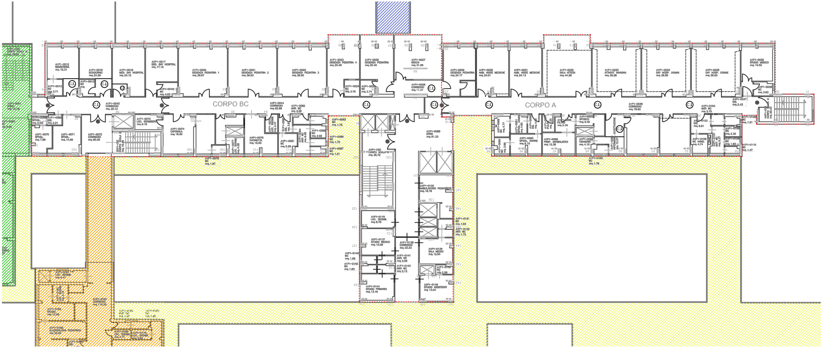
San Camillo de Lellis Hospital



San Camillo de Lellis Hospital



San Camillo de Lellis Hospital



Motivation and Objectives

Performed Tests

Ambient vibration tests

Motivation

- Strategic and complex structure (presence of a joint)
- Seismic vulnerability

Motivation and Objectives

Performed Tests

Ambient vibration tests

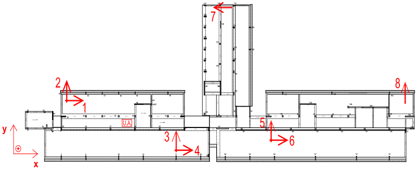
Motivation

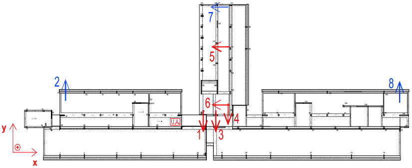
- Strategic and complex structure (presence of a joint)
- Seismic vulnerability

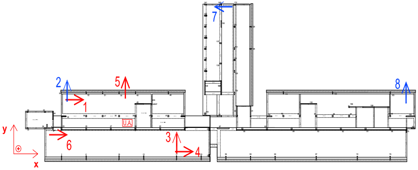
Objective

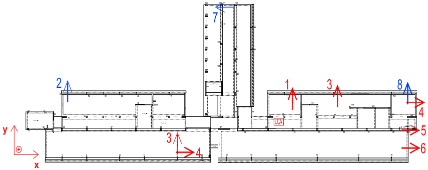
- Characterize the dynamic behaviour
- Evaluate the effectiveness of the joint

Setups



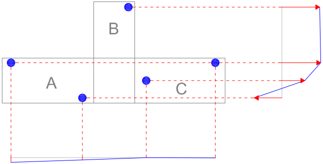






Results

f (Hz)	ξ (%)
2.115	0.91
2.225	0.91
2.405	0.95
2.729	0.86
3.431	0.99



Corp	f (Hz)	ξ (%)
A (setup 1)	2.200	1.45
	2.416	1.44
B (setup 2)	2.230	1.64
	2.428	1.21
	2.518	0.96
	2.945	1.30
	3.638	1.01
C (setup 3)	2.233	1.33
	2.481	1.57
	2.872	1.00
	3.593	1.25

Conclusions

- An output-only identification methodology for linear viscously and non-viscously damped structures has been presented, consisting in three steps: identification of black-box models, definition of modal models and physical models through Finite Element Model Updating (FEMU);
- In the direct problem, the equations of motion have been written in the state-space and the study of the modal properties has been oriented towards the O/O identification;
- Rules to determine the minimum values for the user-defined parameters in Data-SSI have been defined;
- The methodology has resulted effective for each of the case study.

Future Developments

- The implementation of an automated cluster analysis to speed up the process of spurious mode rejection;
- The definition of a model of the structure with the isolators able to cope with different intensities of the excitation;
- Further tests to provide more information about the in-plane behaviour of the footbridge;
- The study of the effects of environmental conditions and adjacent corps on the identification of the structure of the San Camillo de Lellis hospital.